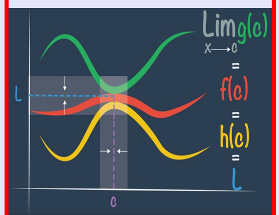
Calculus I

Lecture 45



Feb 19-8:47 AM

Class QZ 20

x-Int & y-Int $(0,0)$
Domain $(-\infty,4)\cup(4,\infty)$
V. A. $x=4$
$\lim_{x\to\pm\infty} f(x)=1 \to$ H. A. $y=1$

Given $f(x)=\dfrac{x}{x-4}$

1) Find $f'(x)$, give x-values where $f'(x)=0$ or undefined

$f'(x)=\dfrac{1(x-4)-x\cdot 1}{(x-4)^2}=\dfrac{-4}{(x-4)^2} < 0$

$f(x)$ is decreasing.

$f'(x)\neq 0$

$f'(x)$ undefined at $x=4$

2) Find $f''(x)$, give x-values where $f''(x)=0$ or undefined.

$f'(x)=-4(x-4)^{-2}$

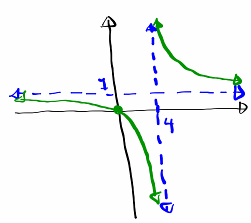
$f''(x)=8(x-4)^{-3}=\dfrac{8}{(x-4)^3}$

$f''(x)\neq 0$

$f''(x)$ undefined at $x=4$

3) Complete the Sign chart.

x	$-\infty$	4	∞
$f'(x)$	$-$	ϕ	$-$
$f''(x)$	$-$	ϕ	$+$
$f(x)$	↘		↘



May 6-9:41 AM

Given $f'(x) = x + \frac{1}{x^3}$, $x > 0$, $f(1) = 6$

Find $f(x)$

$$f'(x) = x + x^{-3}$$

$$f(x) = \frac{x^{1+1}}{1+1} + \frac{x^{-3+1}}{-3+1} + C$$

$$f(x) = \frac{1}{2}x^2 - \frac{1}{2}x^{-2} + C$$

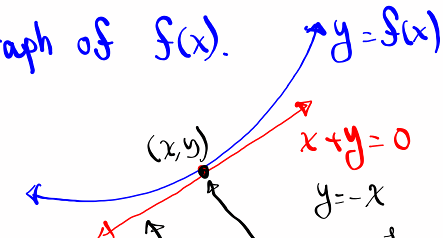
$$f(1) = \cancel{\frac{1}{2}(1)^2} - \cancel{\frac{1}{2}(1)^{-2}} + C = 6$$

$$\boxed{C = 6}$$

$$\boxed{f(x) = \frac{1}{2}x^2 - \frac{1}{2x^2} + 6}$$

May 7-8:53 AM

Find a function $f(x)$ such that $f'(x) = x^3$ and the line $x + y = 0$ is tangent to the graph of $f(x)$.



$f'(x) = x^3$

$f(x) = \frac{x^{3+1}}{3+1} + C$

$\boxed{f(x) = \frac{1}{4}x^4 + C}$

$y = -x$

$m = -1$

$m = f'(x) = -1$

$x^3 = -1$

$\boxed{x = -1}$

$x + y = 0$

$-1 + y = 0 \rightarrow y = 1$

Tangent pt $(-1, 1)$

$f(-1) = 1$

$f(x) = \frac{1}{4}x^4 + C$

$f(-1) = \frac{1}{4}(-1)^4 + C = 1 \rightarrow \boxed{C = \frac{-1}{4}}$

$$\boxed{f(x) = \frac{1}{4}x^4 - \frac{1}{4}}$$

May 7-8:57 AM

Use Newton's method to Solve $\frac{1}{x} = 1 + x^3$.

$\frac{1}{x} = 1 + x^3 \rightarrow x \neq 0 \rightarrow LCD = x$

$1 = x(1 + x^3)$

$1 = x + x^4$

$\underbrace{x^4 + x - 1 = 0}$

$f(x) = x^4 + x - 1$

$f'(x) = 4x^3 + 1$

$$\underbrace{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}_{\text{Newton's Equation}}$$

$$x_{n+1} = x_n - \frac{x_n^4 + x_n - 1}{4x_n^3 + 1}$$

$$= \frac{x_n(4x_n^3 + 1) - (x_n^4 + x_n - 1)}{4x_n^3 + 1}$$

$$= \frac{4x_n^4 + x_n - x_n^4 - x_n + 1}{4x_n^3 + 1} \rightarrow \boxed{x_{n+1} = \frac{3x_n^4 + 1}{4x_n^3 + 1}}$$

Suppose $x_1 = 1$

$$x_2 = \frac{3(1)^4 + 1}{4(1)^3 + 1} = \frac{4}{5} = .8$$

$$x_3 = \frac{3(.8)^4 + 1}{4(.8)^3 + 1} \approx .73$$

$$x_4 = \frac{3(.73)^4 + 1}{4(.73)^3 + 1} \approx .72$$

$$x_5 = \frac{3(.72)^4 + 1}{4(.72)^3 + 1} \approx \boxed{.72}$$

one Solution is $\approx \boxed{.72}$

May 7-9:04 AM

$$f(x) = \frac{x^2}{\sqrt{x+1}}$$

$$f'(x) = \frac{x(3x+4)}{2(x+1)\sqrt{x+1}}$$

$$f''(x) = \frac{3x^2 + 8x + 8}{4(x+1)^2\sqrt{x+1}}$$

Domain $x + 1 > 0$

$x > -1$

$(-1, \infty)$

x-Int & y-Int $(0,0)$

$\lim_{x \to \infty} f(x) = \infty$ NO H.A.

V.A. $x = -1$

$f'(x) = 0 \rightarrow x(3x+4) = 0$

& $x = 0$ & $\cancel{x = -\frac{4}{3}}$

$f'(x)$ is undefined at $x = -1$

$f''(x) = 0 \rightarrow 3x^2 + 8x + 8 = 0$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(3)(8)}}{2(3)} \rightarrow \text{No real Soln.}$$

$f''(x)$ is undefined at $x = -1$

x	-1	0	∞
$f'(x)$			
$f''(x)$			
$f(x)$			

Complete the Sign Chart & graph $f(x)$.

May 7-9:15 AM

Given $f(x) = \dfrac{x^3}{x^2+1}$, $f'(x) = \dfrac{x^2(x+3)}{(x^2+1)^2}$

1) Show $f(x)$ is an odd function.

$f''(x) = \dfrac{2x(3-x^2)}{(x^2+1)^3}$

2) Discuss domain & V.A.

3) Find x-Int & y-Int.

4) Find all x-values where $f'(x)$ & $f''(x)$ are 0 or undefined.

May 7-9:23 AM

Use long division to $\dfrac{x^3}{x^2+1}$

$$\begin{array}{r|l} & x \\ x^2+1 & x^3 + 0x^2 + 0x + 0 \\ & -(x^3 \quad + x \quad) \\ \hline & -x \end{array}$$

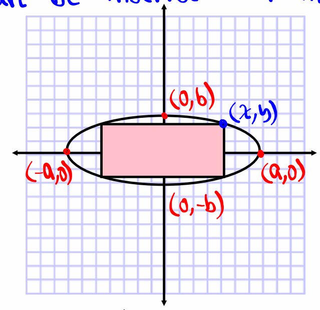
$x^2 \boxed{x} = x^3$

So $\dfrac{x^3}{x^2+1} = x - \dfrac{x}{x^2+1}$ $\quad f(x) = x - \dfrac{x}{x^2+1}$

5) Find $\lim\limits_{x \to \pm\infty} f(x) = \lim\limits_{x \to \pm\infty} \left[x - \dfrac{x}{x^2+1} \right] = \boxed{}$

May 7-9:27 AM

Find the area of the largest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.



Dimensions $2x$ by $2y$

Area $2x \cdot 2y = 4xy$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$LCD = a^2b^2$

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$y^2 = \frac{a^2b^2 - b^2x^2}{a^2}$$

$$y = \sqrt{\frac{b^2(a^2-x^2)}{a^2}}$$

$$y = \frac{b}{a}\sqrt{a^2-x^2}$$

Area $= 4xy$

$$= 4x \cdot \frac{b}{a}\sqrt{a^2-x^2}$$

$$f(x) = \frac{4b}{a} \cdot x\sqrt{a^2-x^2}$$

Maximize $f(x)$

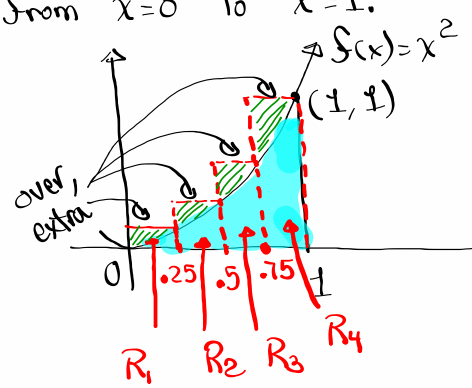
1) $f'(x)$

2) $f'(x)=0$

3) use First derivative test to find x when $f(x)$ has max. value.

May 7-9:33 AM

Find the area below $f(x)=x^2$, above x-axis from $x=0$ to $x=1$.



$A_1 = .25(.25)^2$

$A_2 = .25(.5)^2$

$A_3 = .25(.75)^2$

$A_4 = .25(1)^2$

$$A \approx A_1 + A_2 + A_3 + A_4$$

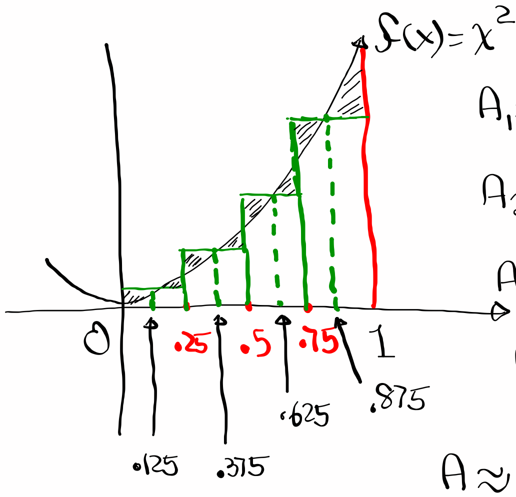
$$\approx .25\left[.25^2 + .5^2 + .75^2 + 1^2\right]$$

$$\approx .46875 \quad \text{over estimate}$$

$$A = \lim_{n\to\infty}$$

$\uparrow$ # of rectangles

May 7-9:42 AM



$A_1 = .25(.125)^2$

$A_2 = .25(.375)^2$

$A_3 = .25(.625)^2$

$A_4 = .25(.875)^2$

$A \approx .25(.125^2 + .375^2 + .625^2 + .875^2)$

$\approx .328125$

Final Ans $A = \frac{1}{3}$

May 7-9:51 AM